MICRO-428: Metrology

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MICRO-428: Metrology

Week Thirteen: Quantum Metrology

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Reference Books

📖 W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015

📖 D.S. Simon, G. Jaeger, A.V. Sergienko, *Quantum Metrology, Imaging and Communication*, 1st ed., 2017

Outline

13.1 Introduction to Quantum Mechanics

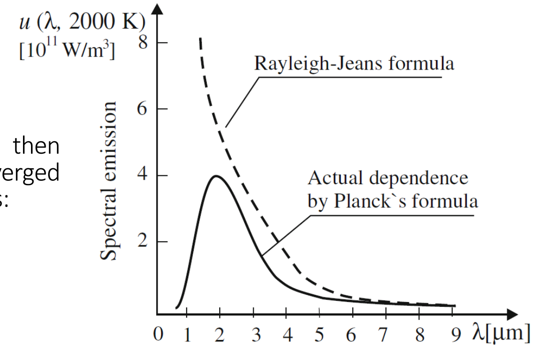
- The development of quantum mechanics was preceded by the discovery by Max Planck (in 1900).

- Planck's assumption was that energy was exchanged in a noncontinuous manner between particles and radiation, and emitted in quanta proportional to a constant ($h = 6.626 \times 10^{-34}$ J s, Planck constant) and the radiation frequency f:

$$E = h \times f$$

- Planck law of spectral emission, based on this assumption, was then formulated, and with respect to the Rayleigh-Jeans formula that diverged at high frequencies, gave more stable and experimentally valid values:

$$u(f,T) = \frac{4hf^3}{c^3} \frac{1}{\exp\left(\frac{hf}{k_B T}\right) - 1}$$



W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.2

13.1 Introduction to Quantum Mechanics

- The quantum mechanics formulation involves the so-called state vectors, which represent possible states of a quantum mechanical system. The use of state vectors allows three novel possibilities that were not available for classical physics:

  - States superposition: if $|\psi_1\rangle$ and $|\psi_2\rangle$ are state vectors representing possible states of the system, then also $\sin\theta\,|\psi_1\rangle + \cos\theta\,|\psi_2\rangle$ is a possible state of the system.

  - Inability to distinguish between nonorthogonal states: suppose that $\langle\psi_1|\psi_2\rangle \neq 0$, then if the incoming state is $|\psi_1\rangle$ there is a nonzero probability to detect $|\psi_2\rangle$ instead. If the measurement is nondestructive, then the measurement itself converts $|\psi_1\rangle$ into $|\psi_2\rangle$.

  - Quantum correlation (*entanglement*): the superposition of two or more states of a multiparticle system. An example of entangled photons is the photon pair created by the positron-electron annihilation due to the beta decay of an atom. That is the fundamental principle underneath the positron-emission tomography (PET).

D.S. Simon, G. Jaeger, A.V. Sergienko, *Quantum Metrology, Imaging and Communication*, 1st ed., 2017, Chap. 1.1

13.1.1 Schrödinger Equation

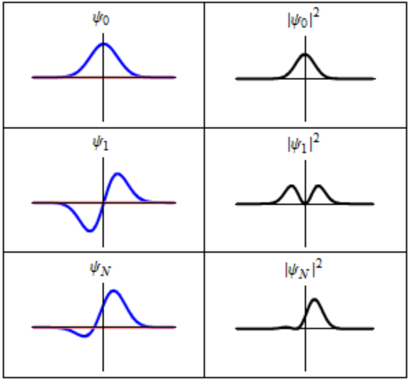
- The Schrödinger equation was formulated in 1926 on the speculative basis by analogy with the then known descriptions of waves and particles.

- The Schrödinger equation describes the state of an elementary particle and features a function referred to as state function or wave function Ψ, and expresses its complex dependence on time and position coordinates of the particles:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + A\Psi = j\hbar\frac{\partial\Psi}{\partial t}$$

- The Schrödinger equation can be also written in terms of the Hamiltonian operator $\hat{H}$, i.e.

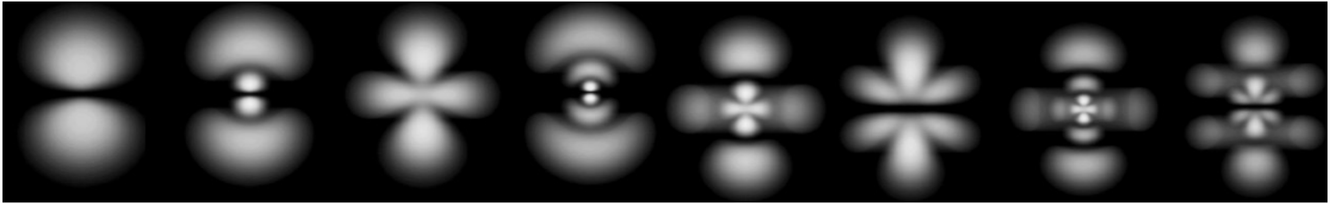
$$\hat{H}|\Psi(t)\rangle = j\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle$$



Wave functions satisfying the time-dependent Schrödinger equation for harmonic oscillations. Source: Wikipedia.

W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.2

13.1.1 Schrödinger Equation



- A Physical interpretation of the wave function was first proposed by Max Born in 1926: the wave function describes the probability that the particle is present within a certain region, specifically in a volume dV. The probability density is proportional to the square of the module of the wave function:

$$P = k|\Psi|^2 dV$$

- Born won the 1954 Nobel Prize in Physics for his "fundamental research in quantum mechanics, especially in the statistical interpretation of the wave function".

W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.2

13.1.2 Heisenberg Uncertainty Principle

- The uncertainty principle is closely related to the particle-wave duality of matter. It defines the limits of accuracy with which the state of a particle can be determined.

- The electron is assigned a wavelength λ, the value of which is related to the momentum of the electron. According to the de Broglie formula:

$$\lambda = \frac{h}{p}, \qquad p = mv$$

- Along the segment Δx, which is the uncertainty associated with the position x of the particle, a wave has n maxima and the same number of minima:

$$\frac{\Delta x}{\lambda} = n$$

W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.3

13.1.2 Heisenberg Uncertainty Principle

- A wave that has zero amplitude beyond Δx, must include waves that have at least $(n+1)$ maxima and minima along this segment:

$$\frac{\Delta x}{\lambda - \Delta\lambda} \geq n+1$$

- From the previous follows:

$$\frac{\Delta x \times \Delta\lambda}{\lambda^2 - \lambda\,\Delta\lambda} \cong \frac{\Delta x \times \Delta\lambda}{\lambda^2} \geq 1$$

- Finally, from the de Broglie formula:

$$\frac{\Delta\lambda}{\lambda^2} = \frac{\Delta p}{h}, \qquad \Delta x \times \Delta p \geq \hbar/2$$

W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.3

13.1.3 Limits of Measurement Resolution

- The limits of measurement resolution are result of limitations coming from:
  - The Heisenberg uncertainty principle;
  - The quantum noise of the measured object;
  - The thermal noise of the measured object.

- The thermal noise power spectral density in an object at an absolute temperature T is described by Planck equation:

$$\frac{P(T,f)}{\Delta f} = hf + \frac{hf}{\exp\left(\frac{hf}{k_B T}\right) - 1}$$

W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.4

13.1.3 Limits of Measurement Resolution

- The Planck equation takes two extreme forms depending on the relationship between the thermal noise energy and the quantum of energy of electromagnetic radiation.

- For $k_B T \gg hf$, it takes the form of the Nyquist formula:

$$E(T) = \frac{P(T)}{\Delta f} \cong k_B T$$

- For $k_B T \ll hf$, it takes into account only the quantum noise:

$$E(T) = \frac{P(T)}{\Delta f} \cong hf$$

k_B= 1.38×10^{-23}JK^{-1}: Boltzmann constant
h= 6.62×10^{-34} m^{2}kg/s: Planck's constant

W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 1.4

Outline

13.2 The Quantum SI

- The Quantum SI has the peculiarity to focus the attention on the universal constants to build the metrology system. Assigning the value of the universal constants will allow all the scaling of all the measures accordingly:

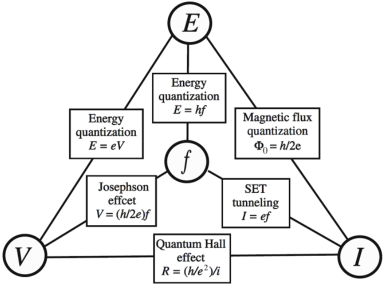
  - Ground state hyperfine splitting transition frequency of Caesium, $\Delta\nu\ ({}^{133}Cs)_{hfs}$ is $9\ 192\ 631\ 770\ \mathrm{Hz}$,
  - Speed of light in vacuum c_0 is $299\ 792\ 458\ \mathrm{m\ s^{-1}}$,
  - Planck constant h is $6.626\ 0693 \times 10^{-34}\ \mathrm{J/s}$,
  - Elementary charge e is $1.602\ 176\ 53 \times 10^{-19}\,\mathrm{C}$,
  - Boltzmann constant k_B is $1.380\ 6505 \times 10^{-23}\ \mathrm{J/K}$,
  - Avogadro constant N_A is $6.022\ 1415 \times 10^{23}$ per mole,
  - Spectral luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}\ \mathrm{Hz}\ K(\lambda_{555})$ is $683\ \mathrm{lm/W}$.

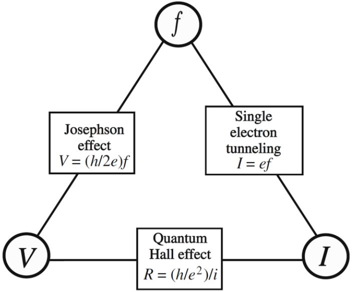
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 3.2

13.2.1 Quantum Metrology Triangle

- The vertices of the quantum metrological triangle are physical quantities: voltage (V), electric current (I), and frequency (f), coupled by three quantum effects, which represent the sides of the triangle.

- These quantum effects are:
  - Josephson Effect,
  - Quantum Hall Effect (QHE),
  - Single Electron Tunneling (SET).





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Outline

13.3.1 Superconductivity

- Quantum voltage standards use the effect of voltage quantization in Josephson junctions. A necessary condition for a Josephson junction to operate is the superconducting state of its electrodes.

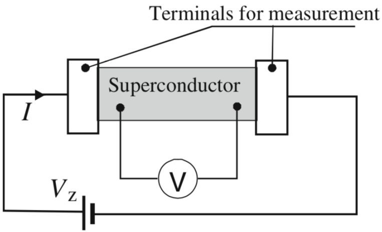
- The transition to the superconducting phase occurs at a specific temperature referred to as critical temperature T_c.

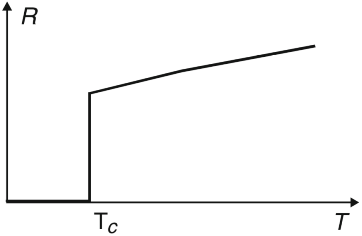
- Common superconducting materials are:

  - Metals: Mercury (1911, $T_c = 4.15$ K), Niobium ($T_c = 9.3$ K), Lead ($T_c = 7.2$ K), Vanadium ($T_c = 5.4$ K)

  - Binary Alloys: Nb_3Ge ($T_c = 23.2$ K), V_3Si ($T_c = 17.1$ K), MgB_2 (2001, $T_c = 39$ K)

  - Ceramics: $La_2Sr_4CuO_4$ (1986, $T_c = 36$ K), $YBaCu_3O_7$ ($T_c = 91$ K).





W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 4.1

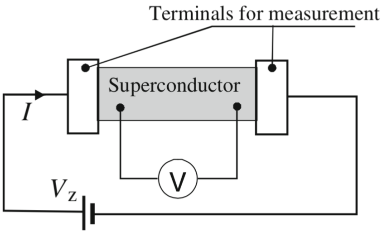
13.3.1 Superconductivity

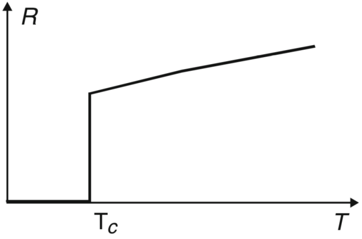
- In 1957 Leon Cooper (USA), a Ph.D. student of John Bardeen (a co-inventor of transistor), proposed to explain the zero electrical resistance in superconductors by the occurrence of electron pairs (referred to as Cooper pairs) in the superconducting phase.

- Each Cooper pair includes two strongly correlated electrons with opposite orientation of momentum and spin. In a superconducting material Cooper pairs form a coherent quantum condensate (fluid) and are all described by a single wave function fulfilling the Schrödinger equation.

- Setting in motion (e.g. that producing an electric current) one electron from a Cooper pair brings about a response of the other electron. As this response involves no energy loss, a number of electrons can move without dissipating any energy to produce a zero-resistance electric current.

- Developed by Bardeen, Cooper and Schrieffer, the theory based on Cooper pairs is commonly referred to as the BCS theory.





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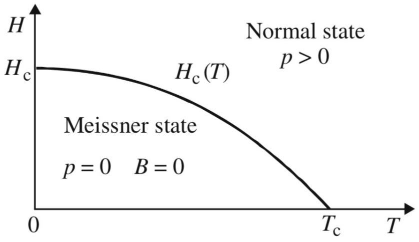
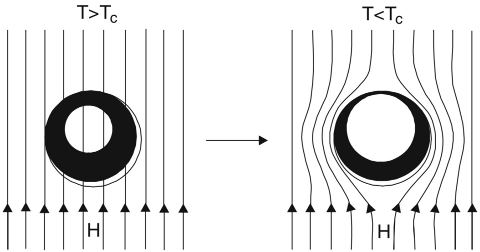
13.3.1 Superconductivity

- In a sufficiently strong magnetic field the superconductor will quit the superconducting state to return to its normal state.
- The minimum value of magnetic field necessary to destroy superconductivity is known as the critical magnetic field H_c. The critical magnetic field H_c increases with decreasing temperature of the sample.
- Another unusual property of superconductors, discovered in 1933 by German physicists Walter Meissner and Robert Ochsenfeld, is the expulsion of magnetic field from the interior of superconductors, which therefore act as ideal diamagnetic materials. This expulsion of magnetic field by superconductors is referred to as the Meissner effect.





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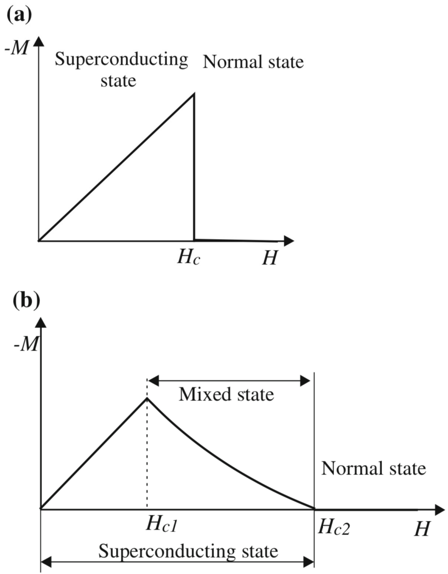
13.3.1 Superconductivity

- The effect of magnetic field on superconducting materials is a criterion of dividing superconductors into two classes: type I superconductors, in which the effect of magnetic field is as shown in the top figure, and type II superconductors, with magnetization dependence as in the bottom figure.

- List of common Type I and Type II superconductors:

  - Type I: Al, In, Mo, Pb, Ta, Ti, W, TiN

  - Type II: Diamond:B, Nb, V, MgB_2, Nb_3Ge, NbN, NbTi, YBCO



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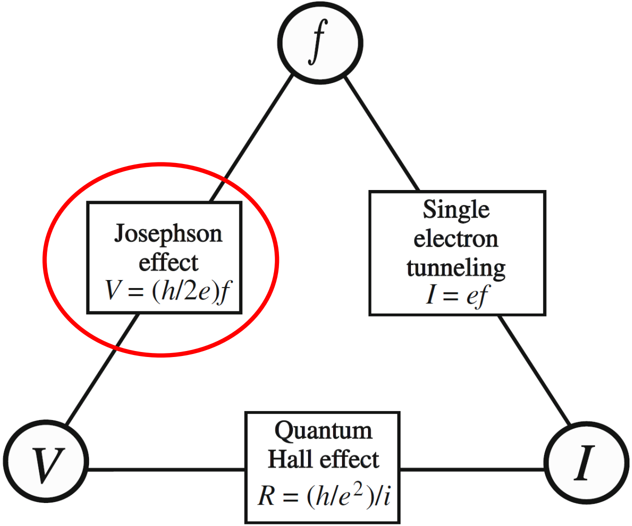
13.3.2 Josephson Effect



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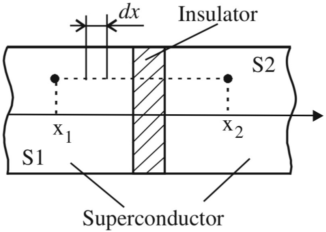
13.3.2 Josephson Effect

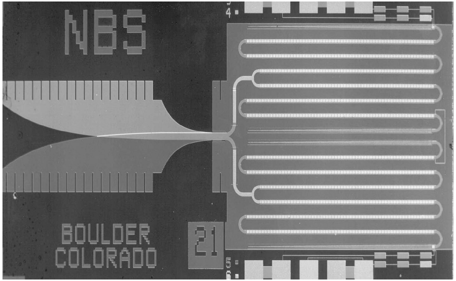
- In 1962 Brian D. Josephson, 22 years old graduate student in Cambridge, studied a junction consisting of two superconductors separated by an insulating layer with 0.1–1 nm thickness.

- Like other elementary particles, Cooper pairs in a superconductor are described within the framework of quantum mechanics by the wave equation

$$\Psi = \Psi_0 \exp\left[-\frac{j}{\hbar}(Et - \boldsymbol{p}\boldsymbol{x})\right], \qquad \rho = \Psi \times \Psi^* = \Psi_0^2$$

where E is the energy of the particle, $\boldsymbol{p}$ and $\boldsymbol{x}$ represent the momentum and position vectors, t is time and ρ is the density of Cooper pairs, according to the Ginzburg-Landau theory.





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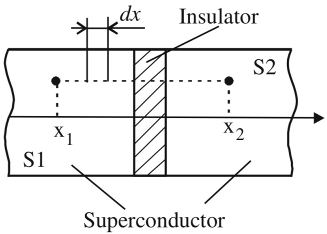
13.3.2 Josephson Effect

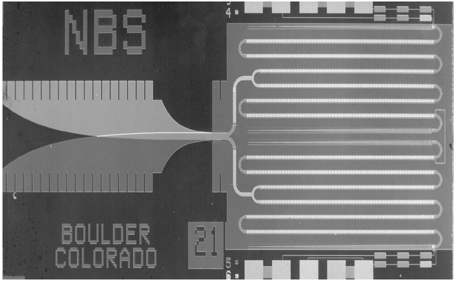
- The wave functions of Cooper pairs in two separated superconductors S1 and S2 are independent. However, if the distance between the superconductors is small (0.1–1 nm), Cooper pairs cross the potential barrier between superconductor S1 and superconductor S2 due to the tunneling effect.



- Let Ψ_1 and Ψ_2 be the wave functions of Cooper pairs in superconductors S1 and S2, respectively:

$$\Psi_1 = \Psi_{01} \exp[-j(\omega t - \varphi_1)], \qquad \Psi_2 = \Psi_{02} \exp[-j(\omega t - \varphi_2)]$$

where $\omega = 2\pi f$ is the angular frequency and φ_i is the phase of Ψ_i



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13.3.2 Josephson Effect

- The derivative of the wave function fulfils the relation:

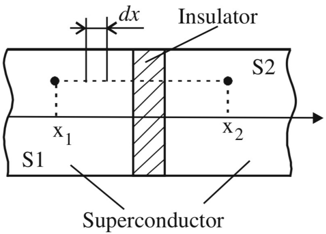
$$\frac{\partial \Psi_1}{\partial t} = -j\frac{E_1}{\hbar}\Psi_1, \qquad \frac{\partial \Psi_2}{\partial t} = -j\frac{E_2}{\hbar}\Psi_2,$$

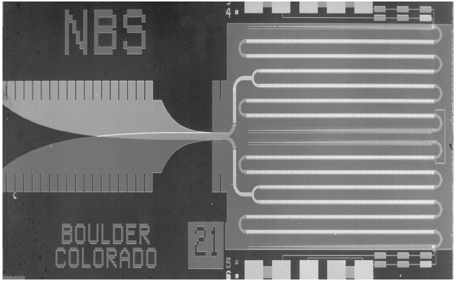
- When the distance between the superconductors is small, their wave functions are correlated due to Cooper pair exchange. The correlation of both wave functions can be expressed:

$$\frac{\partial \Psi_1}{\partial t} = -j\frac{1}{\hbar}(E_1\Psi_1 + K_s\Psi_2),$$

$$\frac{\partial \Psi_2}{\partial t} = -j\frac{1}{\hbar}(E_2\Psi_2 + K_s\Psi_1),$$

where K_s is the superconductor coupling coefficient





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13.3.2 Josephson Effect

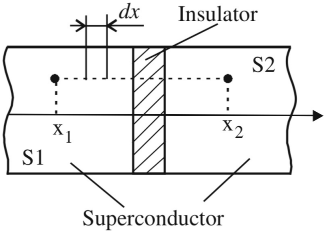
- After some simple manipulations, by equating the real part of the equations, we obtain:

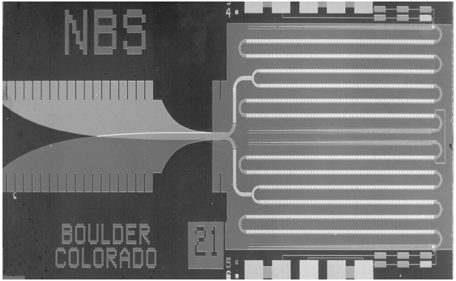
$$\frac{\partial \Psi_{01}}{\partial t} = K_s \frac{\Psi_{02}}{\hbar} \sin(\varphi_2 - \varphi_1)$$

$$\frac{\partial \Psi_{02}}{\partial t} = -K_s \frac{\Psi_{01}}{\hbar} \sin(\varphi_2 - \varphi_1)$$

- Hence, since $\Psi \times \Psi^* = \Psi_0^2$ is proportional to the density of Cooper pairs, the differential $\frac{\partial \Psi_0}{\partial t}$ is proportional to the change in density of Cooper pairs due to tunneling. For this reason:

$$\frac{\partial \Psi_{01}}{\partial t} = -\frac{\partial \Psi_{02}}{\partial t}$$





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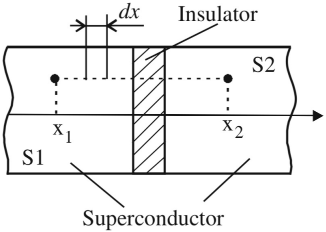
13.3.2 Josephson Effect

- When the superconductors are of the same material this brings to the first Josephson equation:

$$\frac{\partial \Psi_0}{\partial t} = i_s, \qquad i_s = K_s \frac{\Psi_{02}}{\hbar} \sin(\varphi_2 - \varphi_1) = I_C \sin(\varphi_2 - \varphi_1)$$

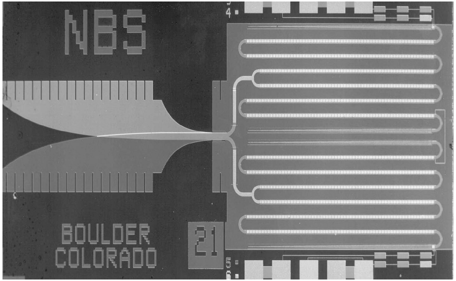
where I_C is defined as critical current of the Josephson junction.



- The oscillating part of the voltage change is given by the imaginary part of the previous equations. By equating them we obtain the second Josephson equation:

$$f = \frac{\partial \varphi}{\partial t} = \frac{2e}{h} V$$

where V is the voltage across the superconducting junction.



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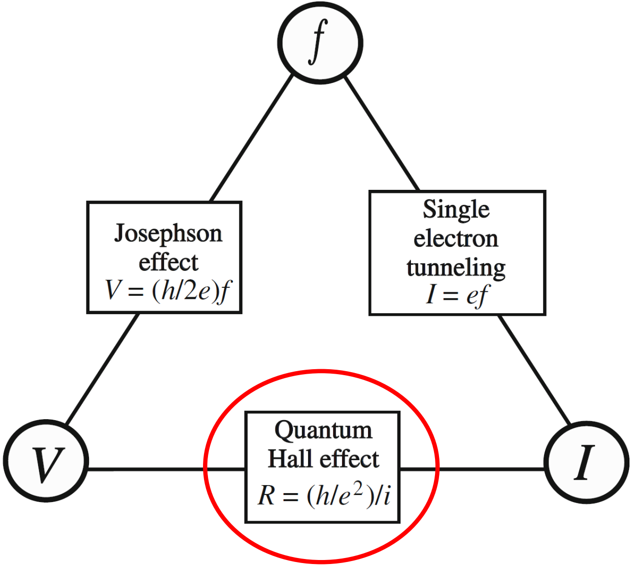
Outline

13.4 Quantum Hall Effect



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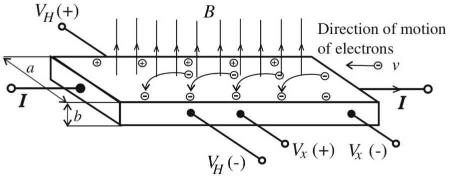
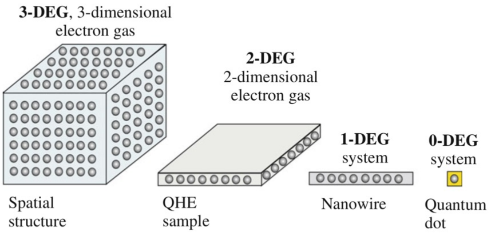
13.4 Quantum Hall Effect

- The quantum Hall effect (QHE) is observed as quantized (stepwise) change in the resistance of a sample as a function of the applied magnetic induction at a temperature of a few kelvins or lower. The QHE is observed in samples with two-dimensional electron gas (2-DEG).
  - A macroscopic sample in which electrons move freely in the three spatial directions, forming a 3-dimensional electron gas (3-DEG).
  - A sample in which the motion of electrons is only possible in two dimensions. Thus, the electrons in the sample form a 2-dimensional electron gas (2-DEG)
  - A sample in which electron transport is only possible in one dimension, along the sample. The electrons in the sample form a 1-dimensional electron gas (1-DEG)
  - A quantum dot, which is a zero-dimensional system (a 0-DEG sample)





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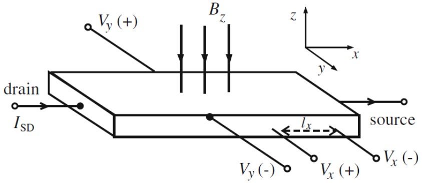
13.4 Quantum Hall Effect

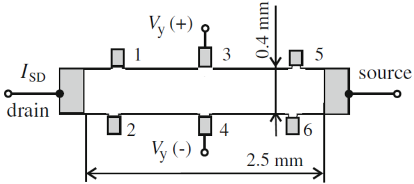
- Quantization of the Hall resistance takes place when a strong magnetic field with a magnetic induction of a few Tesla is applied in the direction perpendicular to the surface of a heterostructure junction within the sample.

- By the rules of quantum mechanics only some trajectories (orbits) are allowed to the motion of electrons, just as only some trajectories are available to an electron traveling on a circular orbit around the nucleus of an atom. The energy levels available to the motion of free electrons are known as the Landau levels.

- The spacing ΔE_L between the Landau levels in an energy band is equal throughout the band and depends on the magnetic induction B:

$$\Delta E_L = \frac{heB}{4\pi m}$$





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13.4.1 Quantum Hall Resistance

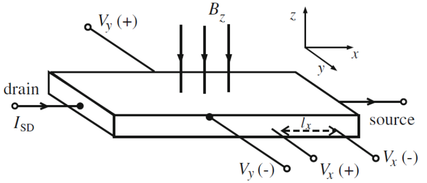
- If the magnetic induction B_z is low, the electrons occupy states in a continuous manner up to the Fermi level, and move freely in the xy plane. Under these conditions, the Hall resistance R_H, given by the formula below, is independent of the size of the sample:

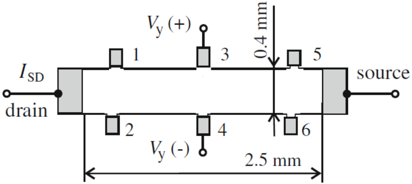
$$R_H = \frac{V_{y1} - V_{y2}}{I_{SD}} = \frac{B_z}{N_s e}$$

where N_s is the surface concentration of electrons.

- A characteristic parameter of a device with two-dimensional electron gas, the degeneracy d, is the maximum number of electrons that can occupy each of the Landau levels. The degeneracy depends on the magnetic induction B and the physical constants e and h:

$$d = \frac{eB}{h}$$





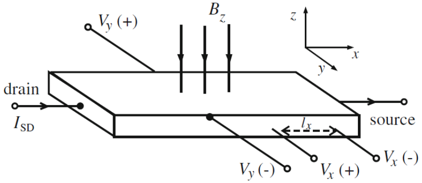
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 6.2

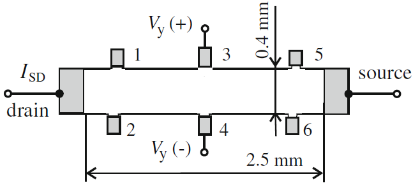
13.4.1 Quantum Hall Resistance

- Let us consider a 2-DEG sample with a surface concentration N_s of free electrons. Elementary particles strive to occupy states at the lowest available energy levels. If the magnetic induction $B_1 = hN_s/e$ is large enough, all the electrons occupy the lowest energy level. The Hall resistance in such a state is determined by:

$$R_H = \frac{B_1}{N_s e} = \frac{hN_s}{e}\frac{1}{N_s e} = \frac{h}{e^2} = R_K$$

where $R_K = 25812.807 \times (1 \pm 2 \times 10^{-7})\,\Omega$ is the von Klitzing constant.





- If the magnetic induction is reduced to $B_2 < B_1$, by the previous formula the degeneracy d will decrease as well. Consequently, the number of states available to electrons at every energy level, including the lowest one, will be lower.

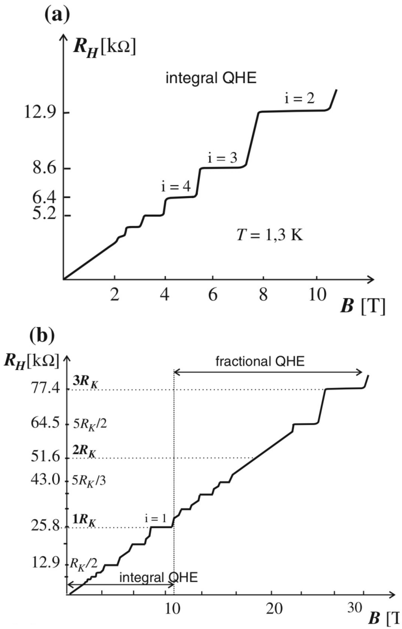
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 6.2

13.4.1 Quantum Hall Resistance

- Further decrease in the degeneracy d implies that successive Landau levels will become available to free electrons. For a magnetic induction corresponding to i energy levels available to electrons the Hall resistance is:

$$R_H(i) = \frac{h}{ie^2} = \frac{R_K}{i}$$

- Surprising results were obtained in studies of the quantum Hall effect in samples cooled to a temperature below 1 K when the magnetic induction was increased to exceed substantially the value corresponding to the first quantization step $(i = 1)$ of the Hall resistance. The explanation of the fractional quantum Hall effect is based on the formation of a new phase, referred to as a quantum liquid. Correlated electrons in the quantum liquid exhibit an electric charge that represents a fraction of the elementary charge e, e.g. $e/3$ or $e/5$.



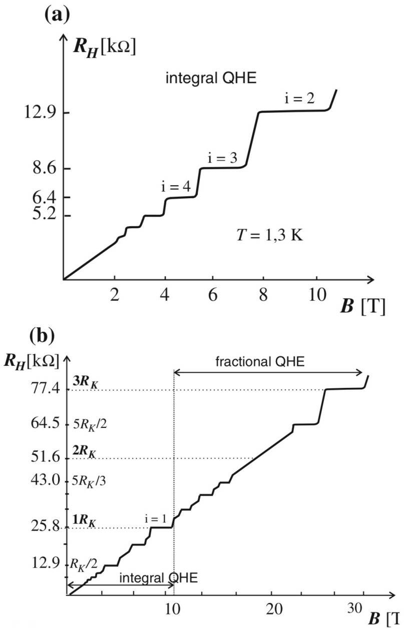
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 6.2

13.4.1 Quantum Hall Resistance

- Robert Laughlin proposed an interesting physical interpretation of the Hall resistance in both the integral and fractional quantum Hall effects. His interpretation is inferred from the physical conditions necessary for the occurrence of the quantum Hall effect, specifically the interaction of free electrons with the magnetic field (in the vortex form).

$$R_H = \frac{h}{ie^2} = \frac{2(h/2e)}{ie} = \frac{2\Phi_0}{ie}$$

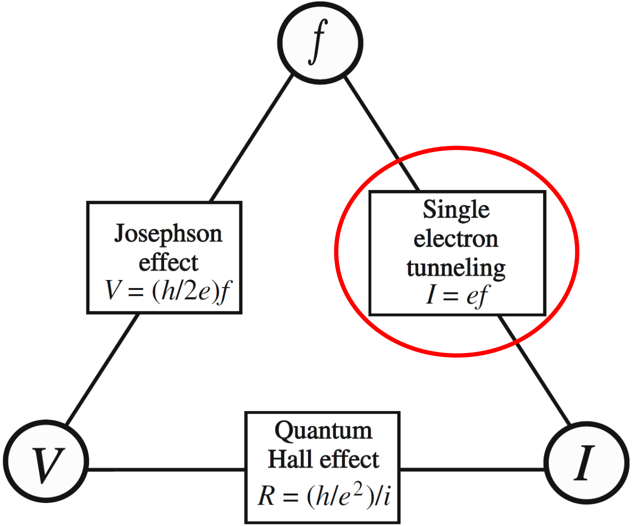
- Laughlin pointed out that the coefficient R_H represents the ratio of the magnetic flux quantum $\Phi_0 = h/2e$ to the electron charge e. The Hall resistance is simply a coefficient representing the ratio of the transverse voltage to the current along the sample, and only for this reason its value is expressed in ohms. In the quantum Hall effect the magnetic flux interacts with fractions of the elementary charge.



W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 6.2

Outline

13.5 Single Electron Tunneling



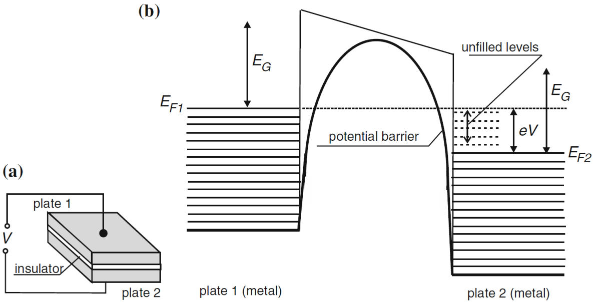
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.1

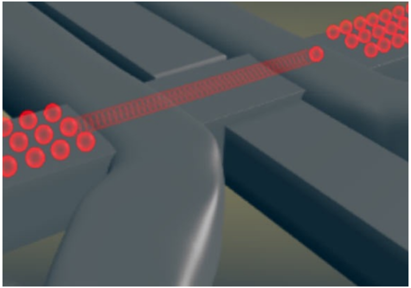
13.5 Single Electron Tunneling

- Charge carriers tunneling through the potential barrier is one of the phenomena at the basis of quantum mechanics. Two metal plates separated by the insulating layer form a thin tunnel junction, which is a system where electron tunneling occurs.

- Current flow, i.e. transport of electrons between two plates (electrodes), requires the performance of work E_G electrons (electron work function of metal) and overcome the potential barrier (energy gap) formed by the insulating layer.

- A variation of the above-described phenomenon of tunneling is the tunneling of single electrons—SET (Single Electron Tunneling). The ability to monitor or control the movement of single electrons in a conductor is associated with the overcome by single electrons of the Coulomb blockade.





W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.1

13.5 Single Electron Tunneling

- The energy required to charge a sample by one additional electron faces a significant obstacle to the transfer—the Coulomb blockade energy.

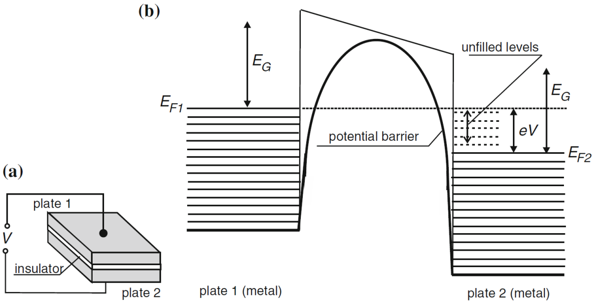
- The Coulomb blockade energy of such a SET junction is described by the formula:

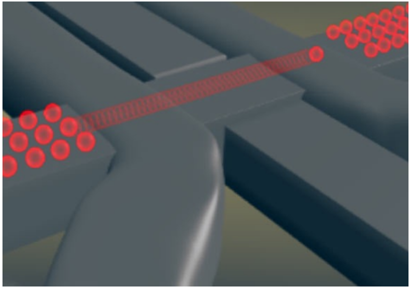
$$E_C = \frac{e^2}{2C_T}$$

where C_T is the capacitance of the tunnel junction.

- It is important to compare E_C with other types of energy in the system: heat, radiation, and so on. E_C has to be much greater than the thermal energy if we want to observe or to measure it:

$$E_C \gg k_B T$$

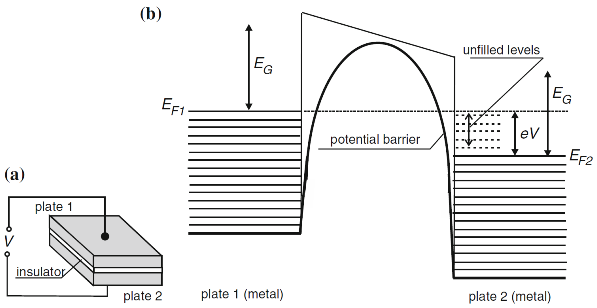




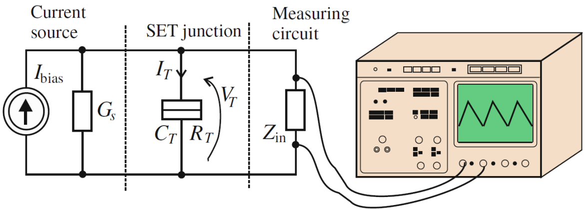
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.1

13.5 Single Electron Tunneling

- Insulation separating the metal plates in the SET junction should ensure a sufficiently high resistance R_T of the junction. R_T value is compared with the von Klitzing constant $R_K = h/e^2 \approx 25.8\,k\Omega$; $R_T > R_K$. High resistance of the junction is necessary for tunneling of electrons ($1e$) through it. If the resistance of the junction is not greater than R_K, that electrons are not well localized to the junction electrodes, and consequently the quantum fluctuations of electrical charge suppress the Coulomb blockade.



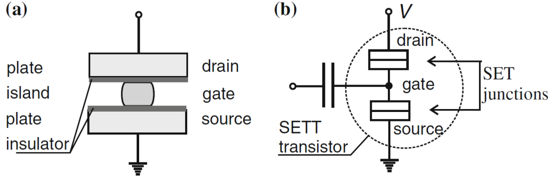
- A SET junction is the source of the measurement signal and the internal resistance of the source, R_T, should be significantly smaller than the input impedance of the measuring circuit Z_{in}, for example an input circuit of oscilloscope. Resistance R_T of SET junctions should be in the from 100 kΩ to 1 MΩ, which together with the capacitance values of C_T (from 0.1 to 1 fF) yields a time constant: $R_T C_T$ from 10 ps to 1 ns.



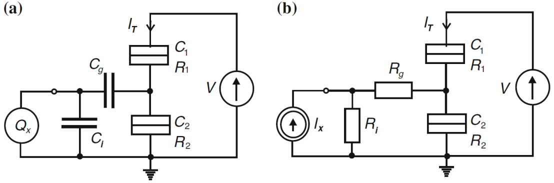
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.1

13.5.1 SETT Transistor

- Two tunnel junctions create SETT transistor (Single Electron Tunneling Transistor), composed by two metallic plates and a single metal speck, which is a common component of the two junctions. The speck is called an island. By analogy to the FET transistor the electrodes in SETT are called: drain, source and gate-island.



- If we apply a voltage of 1 mV between the drain and source, the current I_T will flow through the transistor as a stream of sequential tunneling electrons.



- Current I_T is controllable by the charge applied to the gate or by the voltage connected to the gate. Accordingly, we distinguish between the type of transistors controlled by capacitive coupling, and transistors controlled by resistive coupling. Sensitivity of a SETT transistor, $\Delta I_T/\Delta Q$, can reach 10^{-4} e/s, which is possibly 10^{-4} e in band of 1 Hz.

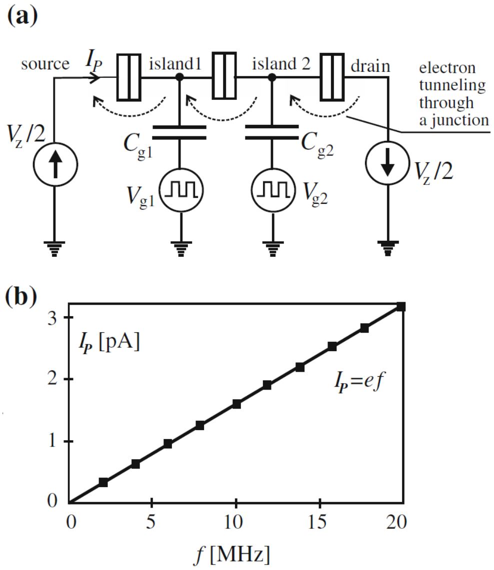
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.2

13.5.2 Electron Pump

- The Electron Pump is an electronic circuit with three junctions to the tunneling of single electrons, including two islands. SET junction matrix may comprise more than three of these junctions.

- The supply voltage V_Z, symmetrical to the ground, is applied to electrodes "drain" and "source". The voltage $V_{g1}(t)$ and $V_{g2}(t)$ controls gate 1 and gate 2, respectively. The voltages are attached to the gates using coupling capacitors (C_{g1} and C_{g2}) of less capacity than the capacity of the SET junction itself.

- After reaching the island the electron is closed in an electrostatic trap there. Until we apply the voltage V_{g2} to the second gate, we will not lower the energy barrier to allow tunneling, and the electron won't tunnel from the island of 1–2.



W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.2

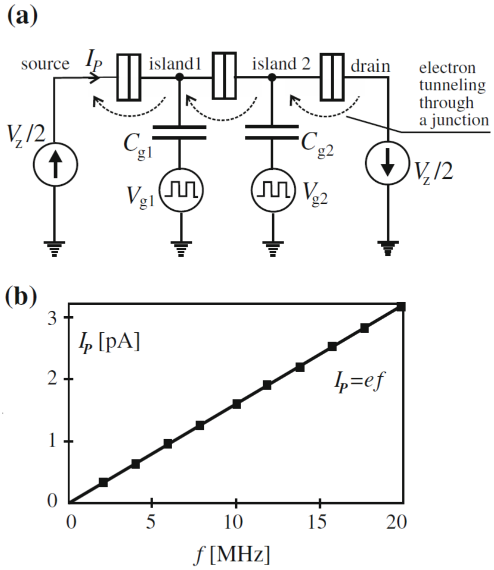
13.5.2 Electron Pump

- The process of opening the gates must be done synchronously, so the signals V_{g1} and V_{g2} should have the same frequency and be out of phase. In this way, it is possible to "pump" one electron from island to island.

- Current I_P in the circuit, flowing as result of electron pumping, may flow in both directions, depending on the order in which the gates are controlled by voltages. The $+I_P$ current requires the pulses sequence: V_{g1} first and V_{g2} second and the $-I_P$ current for the sequence V_{g2} first and V_{g1} second.

- In the electron pump circuit one and only one electron can pass in one cycle:

$$T_{\text{rep}} = \frac{1}{f_{\text{rep}}} \quad \rightarrow \quad \overline{I_P} = e \times f_{\text{rep}}$$



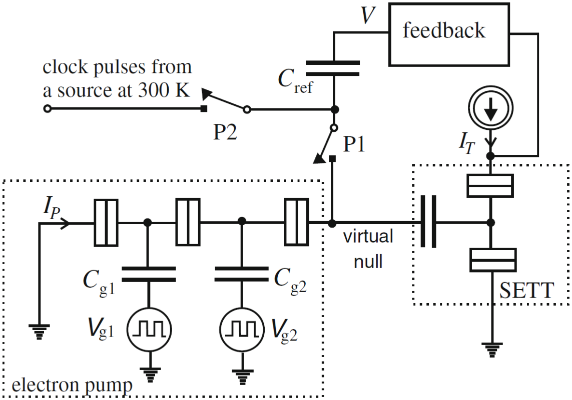
W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.2

13.5.3 Electron-Counting Capacitance Standard (ECCS)

- Instead of using for the current standard construction—the SET pump can be used to build a standard of electrical capacitance: if you are pumping a current of 1 pA to 1 pF capacitor for 1 s, then the voltage on capacitor will be 1 V.

- Three essential parts of the system are: an electron pump, a SETT transistor and a cryogenic capacitor C_{ref} of about 1 pF, cooled together with SET circuits.

- The ECCS system operates in the two phases selected by switching the switches P1 and P2. In the first phase the switch P1 is closed and the switch P2 open, and the pump is pumping electrons on the inner plate of a capacitor C_{ref}. During this process, the voltage along the pump must be kept close to zero, or practically must be zero voltage at the point of "virtual zero". The zero detector (transistor SETT) and the feedback system provide keeping the zero voltage.

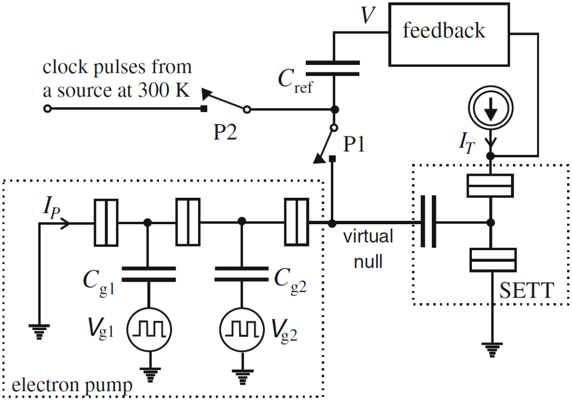


W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.3

13.5.3 Electron-Counting Capacitance Standard (ECCS)

- After pumping about 10^8 electrons in one direction, the pump stops and the voltage V is measured—it should be approximately +10 V. Then the same number of electrons is pumped in the opposite direction and then the voltage is measured it should be approximately −10 V. This process of reload of $C_{\mathbf{ref}}$ capacitor. The process is repeated 10–100 times (each reloading series takes about 100 s) to obtain a one, average value $C_{\mathbf{ref}}$.

- In the second phase of the operation cycle of the ECCS the switch P1 is open and P2 is closed. Then, using an alternating current bridge, the capacitance of the capacitor $C_{\mathbf{ref}}$ is compared to another capacitor placed at room temperature. This allows to determinate the electric charge (and further the capacity) of the last capacitor in an elementary charge units.



W. Nawrocki, *Introduction to Quantum Metrology: Quantum standards and instrumentation*, 1st ed., 2015, Chap 8.3

Outline

13.6 Quantum Applications

- Quantum Imaging:
  - Quantum Ghost Imaging
  - Quantum Image Distillation
  - Positron Emission Tomography

- Quantum computing

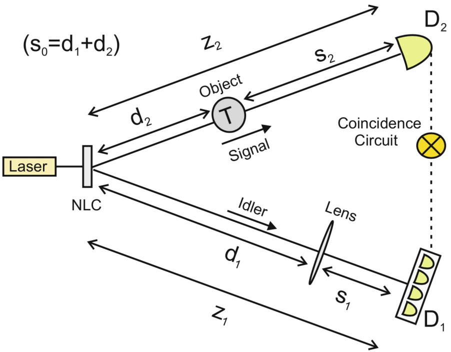
- Quantum Communication:
  - Quantum Key Distribution or Quantum Cryptography
  - Quantum Random Number Generation

13.6.1 Quantum Ghost Imaging

- Ghost imaging, also known as two-photon or correlated-photon imaging, was first demonstrated in the 1990s and quickly gained widespread attention due to the fact that it exhibits a highly non-intuitive effect: an image can be formed by looking at correlations between two beams of light, neither of which is capable of forming an image by itself.

- In the early experiments, this was achieved by means of the non-local correlations present in systems of entangled photon pairs: one photon probes the object but is detected by a detector lacking significant spatial resolution, while the second photon never interacts with the object at all. Neither detector supplies sufficient information to form an image but the image can, nonetheless, be reconstructed by looking at the coincidence counts between the two detectors.

- Correlated imaging has a number of advantages and unusual features. Importantly, two beams may be of different frequency so that the object can be probed at one wavelength but the image formed at a different wavelength.



D.S. Simon, G. Jaeger, A.V. Sergienko, *Quantum Metrology, Imaging and Communication*, 1st ed., 2017, Chap 6

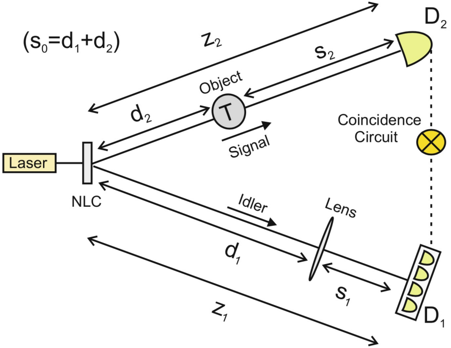
13.6.1 Quantum Ghost Imaging

- A schematic diagram of the quantum ghost imaging apparatus is shown in figure. The use of down-conversion as a light source means that the flux of photons is very low, requiring the use of photon-counting detectors.

- The signal is transmitted through the object to D_2, a single-pixel bucket detector that simply registers whether or not a photon is detected, without determining the location at which it arrived. D_2 registers whether the photons pass through the object or are blocked.

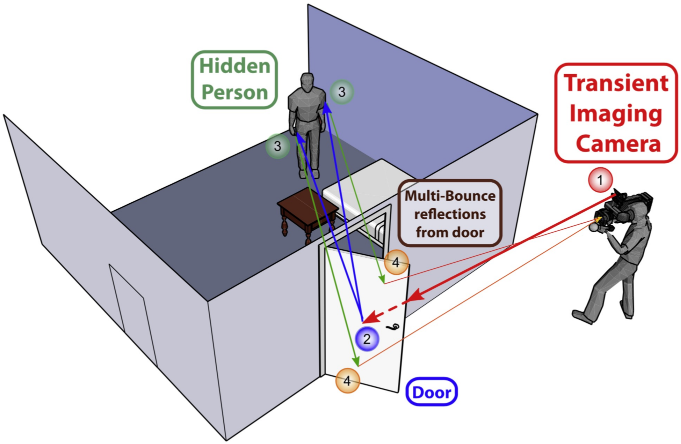
- D_1, on the other hand, is a high spatial-resolution detector—a CCD camera or an array of SPADs. The lens in branch 1 has focal length f. Let d_1 and d_2 be the distances from the nonlinear crystal to the lens and from crystal to object, and let $s_0 = d_1 + d_2$. The distance s_1 is that from the lens to detector D_1. The distances s_0 and s_1 satisfy the imaging condition:

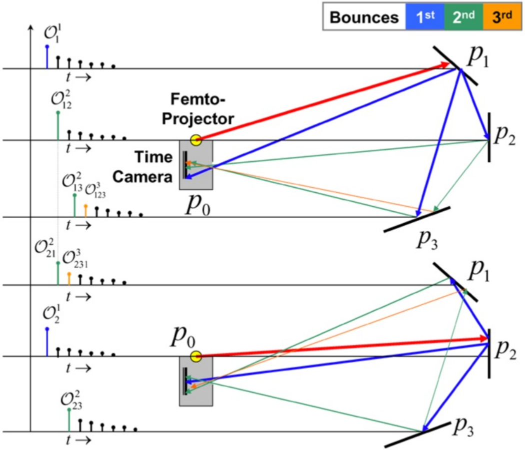
$$\frac{1}{s_1} + \frac{1}{s_0} = \frac{1}{f}$$



D.S. Simon, G. Jaeger, A.V. Sergienko, *Quantum Metrology, Imaging and Communication*, 1st ed., 2017, Chap 6.1

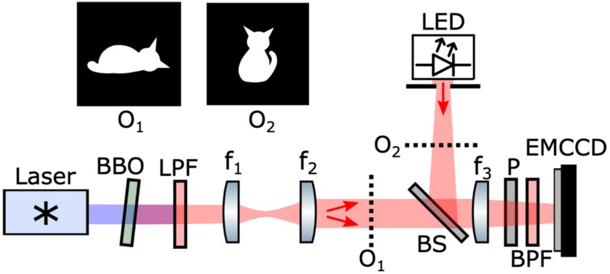
13.6.1 Quantum Ghost Imaging – Around-the-Corner





📖 A. Kirmani, *Looking Around the Corner using Ultrafast Transient Imaging*, International Journal of Computer Vision 95, 2011

13.6.2 Quantum Image Distillation



- Many pairs of photons are generated that are entangled, i.e. knowing the state of one implies knowing that od the other.
- The entangled photons go through object O_1 and continue towards the sensor.
- 'Classical' (non entangled) photons generated in the LED go through object O2 and are mixed with 'quantum' (entangled) photons on the sensor.
- What happens?
  - If pairs and single photons are not distinguishable, then the images will be superposed.
  - If pairs are distinguished from single photons, then the two images can be separated.
- A sensor is required to detect entangled pairs. How?

H. Defienne, M. Reichert, J. W. Fleischer, D. Faccio, *Quantum Image Distillation, ArXiv and Science advances, 2019*

13.6.2 Quantum Image Distillation

- How to detect entangled pairs?
  - Entangled photons are center-symmetrical wrt the laser point (spatial)
  - Entangled photons impinge at the same time (coincident)
- Space: compute the intensity correlation for each pixel and the center-symmetric pixel

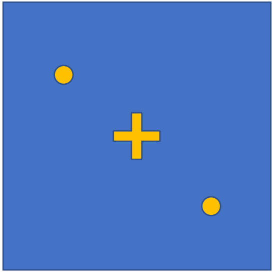
$$\Gamma(r_1, r_2) = \langle I(r_1)I(r_2)\rangle - \langle I(r_1)\rangle\langle I(r_2)\rangle$$

where

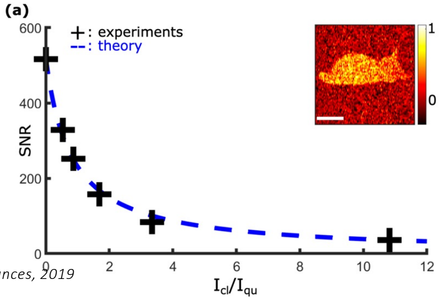
$$\langle I(r_1)I(r_2)\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{l=1}^{N} I_l(r_1)\, I_l(r_2)\,, \langle I(r_1)\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{l=1}^{N} I_l(r_1)$$

l represents the frame index, N is chosen very large.

Spatial detection



- Intensity ratio between classical and quantum photons



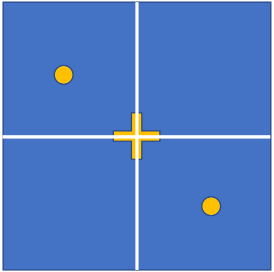
H. Defienne, M. Reichert, J. W. Fleischer, D. Faccio, *Quantum Image Distillation, ArXiv and Science advances, 2019*

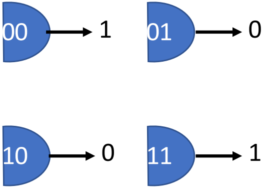
13.6.2 Quantum Image Distillation

- Coincidence: compute the correlation of each pixel with all the surrounding pixels for a given frame.
  - The coincidence resolving time is the frame
  - We use TDCs, the CRT is the LSB of the TDC
- For each pixel check if there is one pixel in anti-symmetric position that has impinged in the same frame. if so, the pair is identified.
- When a photon is detected, check all coincident photons (+/- e certain number of LSBs). If one is found, check if it is anti-symmetric. If so, the pair is identified. If not, it is discarded.

- What is the advantage of using TDCs?

Spatial detection





In a single frame l.

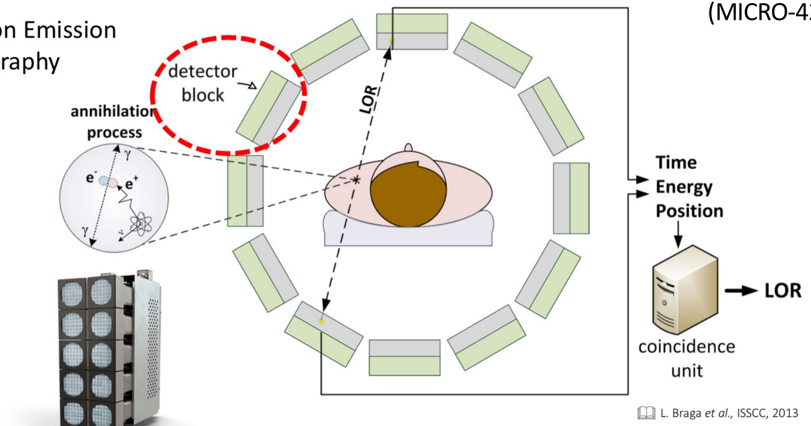
H. Defienne, M. Reichert, J. W. Fleischer, D. Faccio, *Quantum Image Distillation, ArXiv and Science advances, 2019*

13.6.3 Positron Emission Tomography (PET)

- In PET, upon annihilation, the two photons generated by are entangled.
- Coincidence and energy are used to identify pairs.

(MICRO-428 – Week 3)

Positron Emission Tomography Basics

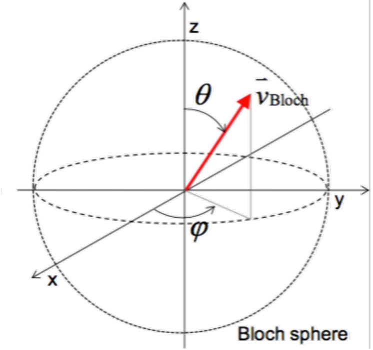


L. Braga *et al.*, ISSCC, 2013

GE Discovery IQ, Nov 2016

13.6.4 Quantum Computing

- In Quantum computing (QC), quantum entanglement and superposition are used to perform computations (Feynman, 1980s)
- In QC, the quantum bit, or qubit, is the basic computational unit.
- Thanks to superposition, 2^N states are simultaneous in n qubits, thus today's intractable problems could be solved with an array of N qubits, with N large.

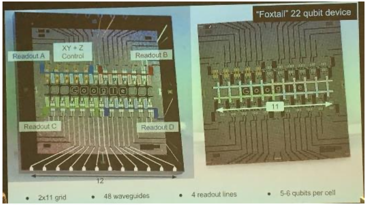


$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$$

Richard Feynman

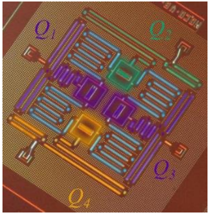
13.6.4 Solid-State Qubit Implementations

72-qubit chip announced



Google

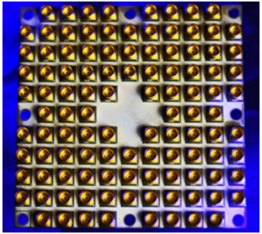
16 Qubits online version



IBM

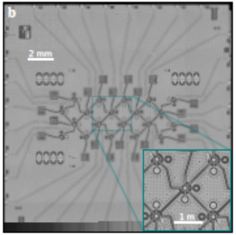
50 qubit chip announced

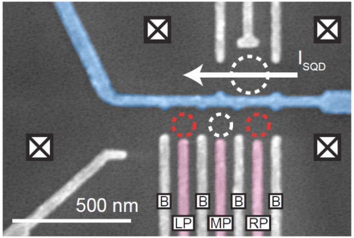
49 qubit chip



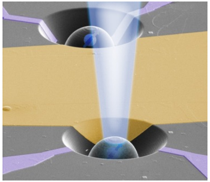
intel

19 qubit chip





Semiconductor quantum dots



Impurities in diamond or silicon

Source: Tristan Meunier & Lieven Vandersypen

13.6.4 Multi-Qubit States

- How to describe two-qubit system in a Bloch sphere?
  - Not possible
  - One needs an alternative representation
  - Try two Bloch spheres!

$$|\Psi\rangle = \alpha_{11}|11\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{00}|00\rangle \doteq \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix}$$

$$\alpha_{00}, \alpha_{01}, \alpha_{10}, \alpha_{11} \in C$$

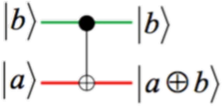
$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1$$

Global phase is not relevant.

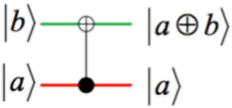
13.6.4 two-Qubit Gates

- With 2 or more qubits, we can construct complex gates
- With gates, we can construct quantum circuits
- Quantum circuits can be mapped to quantum algorithms

Controlled-Not gates

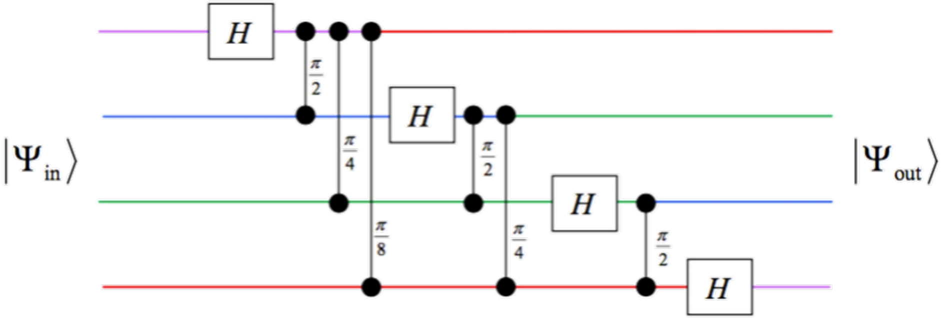


$$\begin{matrix} & |00\rangle & & & |11\rangle & \\ \text{C-NOT}_{gr} \doteq & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{matrix} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{matrix} \end{matrix}$$

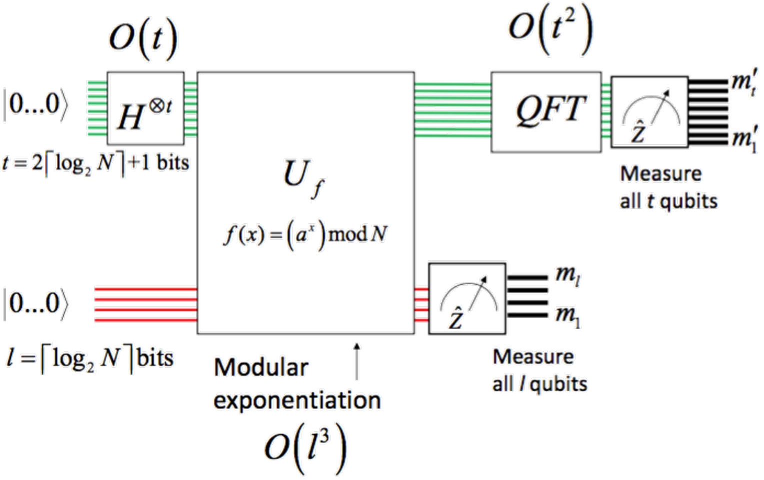


$$\text{C-NOT}_{rg} \doteq \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

13.6.4 Quantum Circuits



13.6.4 Quantum Algorithms

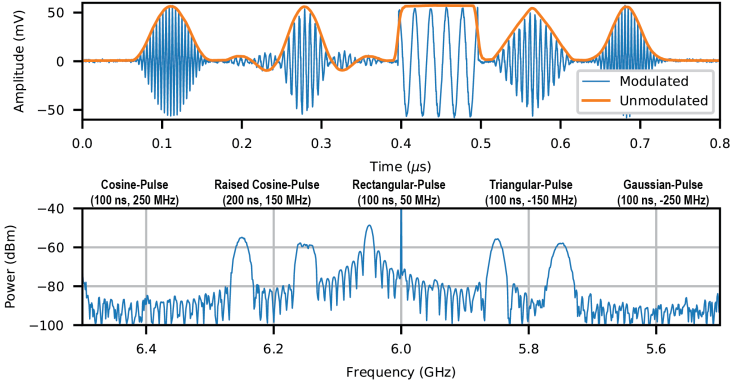


13.6.4 Measuring Qubits

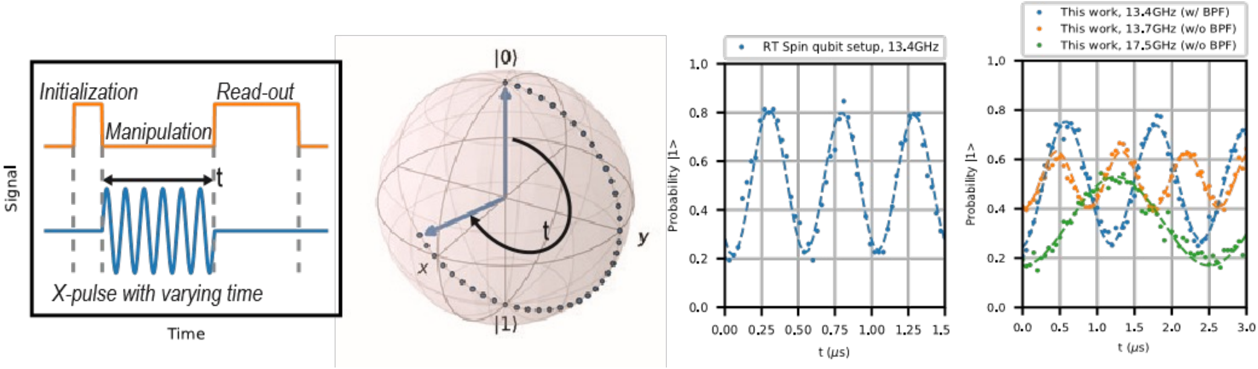
- In principle, qubits are measured only at the end of the execution of the quantum algorithm
- Though, quantum error correction is required to fix deviations of the state of a qubit
- Qubits are measured
  - Electrically, through reflectometry or capacitively
  - Optically, through single-photon detection
- Example of electrical stimulation of qubits

  for measurement

B. Patra *et al.*, *ISSCC* 2020

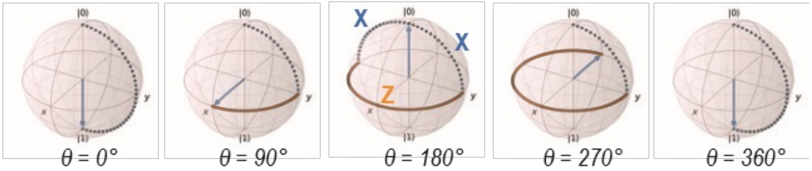


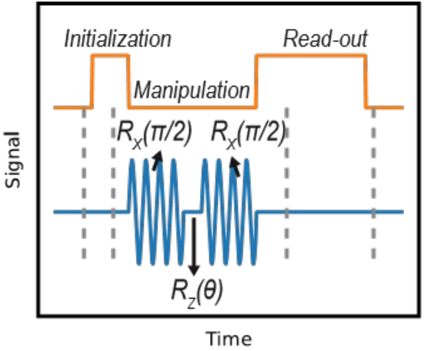
13.6.4 Measuring Qubits: Rabi Oscillation

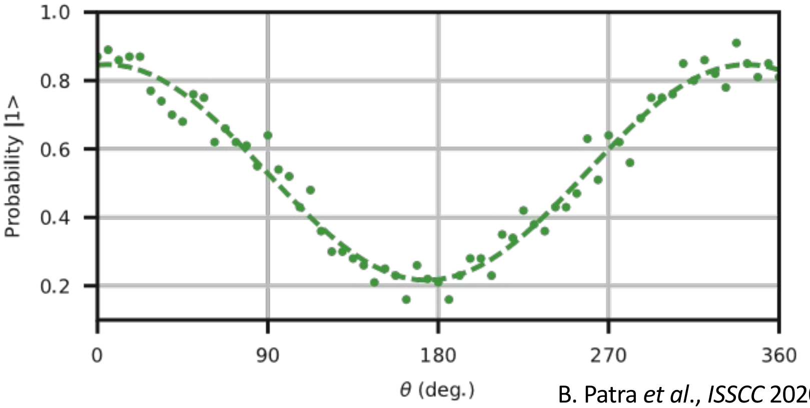


B. Patra *et al.*, *ISSCC* 2020

13.6.4 Measuring Qubits: Multiple rotations



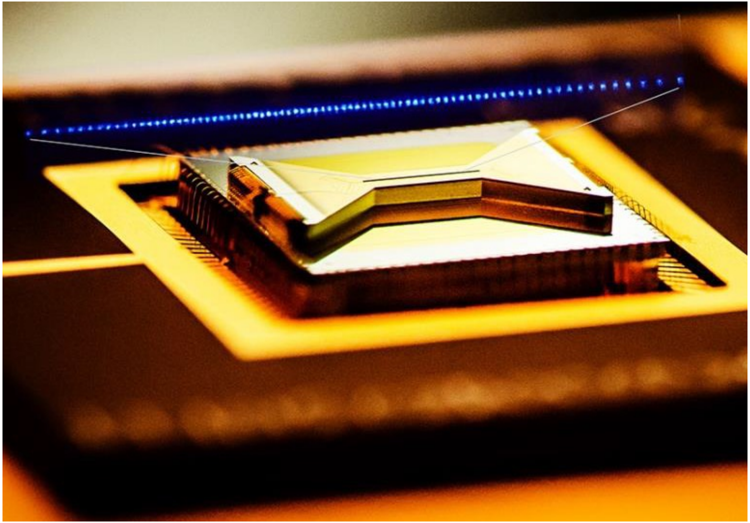




B. Patra *et al.*, *ISSCC* 2020

13.6.4 Measuring Qubits: Optical Measurement

- Ion traps read out optically
- Single-photon detection is required
- Timing resolutions of the order of picoseconds are necessary



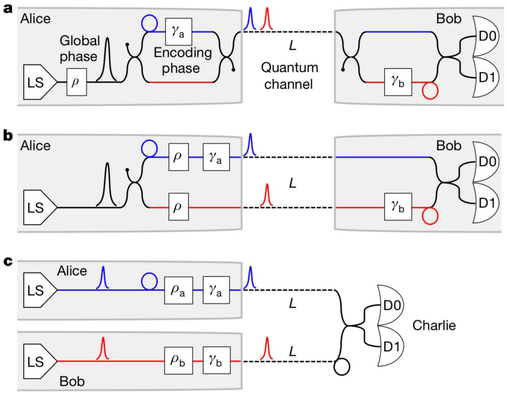
Curthesy: C. Monroe

13.6.5 Quantum Key Distribution

- Despite we would like to mitigate as much as possible the disturbances to an optical system, there are disturbances that are inherent to any quantum system, which can not be eliminated or reduced beyond a fundamental limit; these stem from the Heisenberg uncertainty relations and from the inability to distinguish non-orthogonal states.

- Quantum key distribution (QKD), also known as quantum cryptography, takes advantage of the fact that if an eavesdropper (traditionally known as Eve) tries to read a secret communication sent over a quantum channel, she inevitably introduces disturbances to the signal.

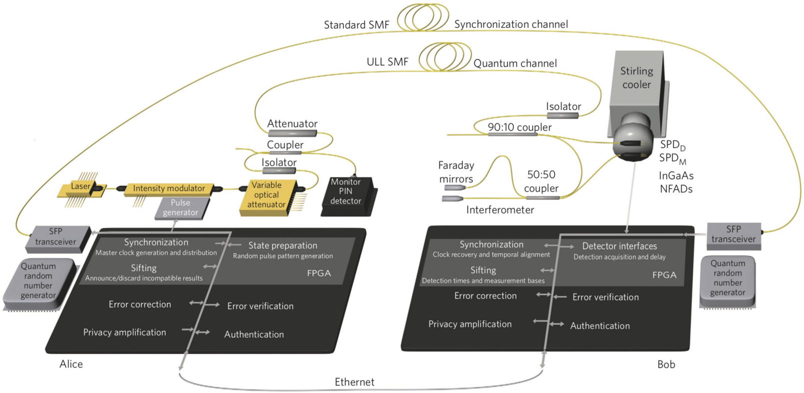
- These disturbances produce measurable errors in the signal which then tip off the legitimate users of the system (Alice and Bob) to the tampering. When the error rates exceed those expected from normal physical noise sources in the system alone, the users avoid possible eavesdropping by shutting down the channel and switching to a different secured key distribution channel to continue their cryptographic task.



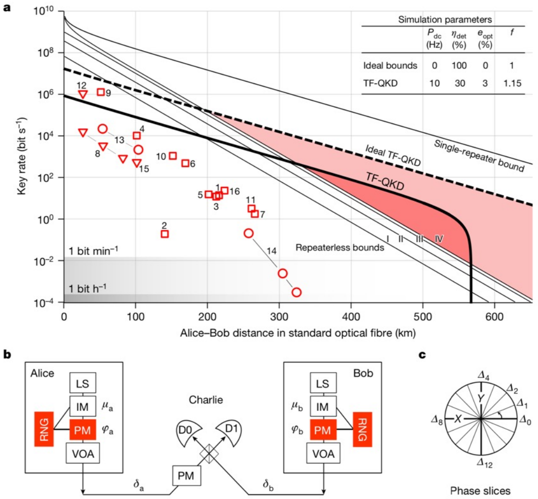
M. Lucamarini et al., *Overcoming the rate–distance limit of quantum key distribution without quantum repeaters*, Nature 557, 2018

D.S. Simon, G. Jaeger, A.V. Sergienko, *Quantum Metrology, Imaging and Communication*, 1st ed., 2017, Chap 9.1

13.6.5 Quantum Key Distribution – Experiment

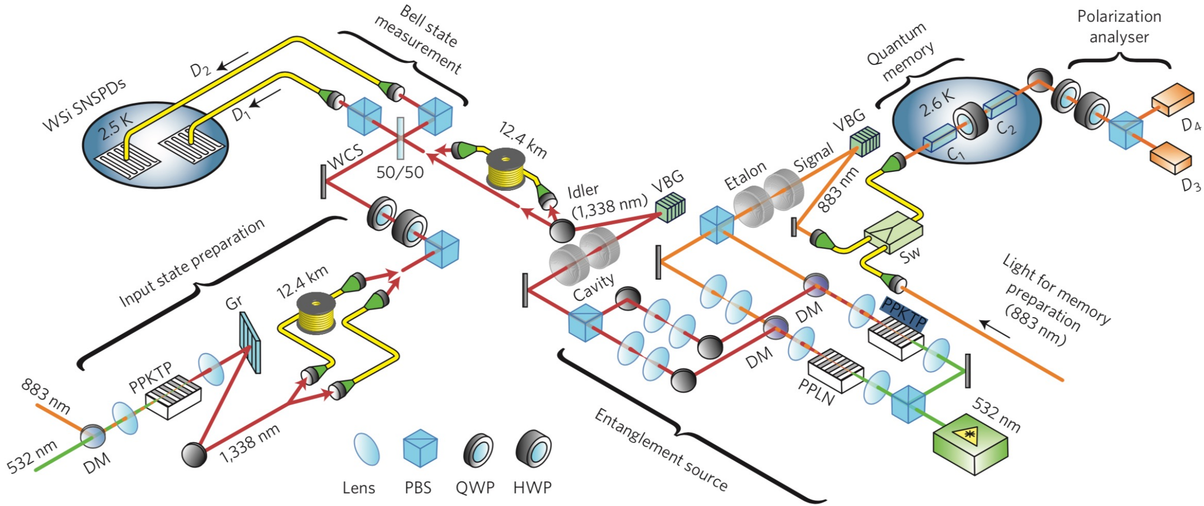


B.A. Korzh et al., *Provably secure and practical quantum key distribution over 307 km of optical fibre*, Nature Photonics 9, 2015



M. Lucamarini et al., *Overcoming the rate–distance limit of quantum key distribution without quantum repeaters,* Nature 557, 2018

13.6.5 Quantum Key Distribution – Quantum Teleportation



F. Bussières et al., *Quantum teleportation from a telecom-wavelength photon to a solid-state quantum memory*, Nature Photonics 8, 2014

13.6.6 Quantum Random Number Generation

- Random numbers are a fundamental resource in science and engineering with important applications in simulation and cryptography. The inherent randomness at the core of quantum mechanics makes quantum systems a perfect source of entropy.

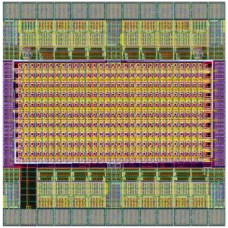
- Defining randomness is a deep philosophical problem.

- In computing, it is important to distinguish between algorithmically generated numbers that mimic the statistics of random distributions and random numbers generated from unpredictable physical events.

- Methods that produce random numbers from a deterministic algorithm are called pseudorandom number generators (PRNGs). While it is clear that any algorithmically generated sequence cannot be truly random, for many applications the appearance of randomness is enough (simple simulations, for example).

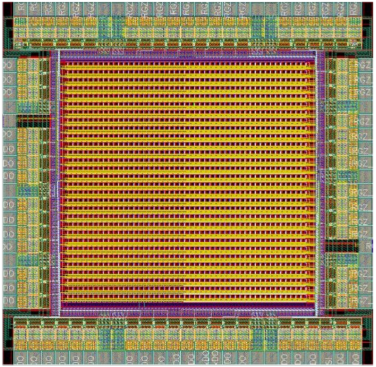
M. Herrero-Collantes, *Quantum Random Number Generators*, Reviews of Modern Physics 89, 2017

13.6.6 Quantum Random Number Generation



- When pseudorandom number generation does not provide enough randomness (in cases such as cryptography or very complex Monte Carlo simulations), the use of a quantum random number generator (QRNG) is required.

- One approach is presented by Burri *et al*. The presented quantum random number generator is based on a matrix of single-photon avalanche diodes (SPADs) illuminated by a low intensity source of photons. During the propagation between source and detectors, the photon behaves as a wave and is described by a wave-function. Then, the photon will collapse randomly in the image plane, where the SPADs are located. Thus, the SPADs in the array will randomly click and generate a raw bit sequence.



- Another approach might be one where, instead of relying on actual photon-driven clicks of the detector, the dark count rate is playing the central role. The DCR is indeed intrinsically more entropic than flood illumination, and would give an even more randomized QRNG. However, for this approach, the maximum count rate could result an issue.

D. Stucki, *Towards a high-speed quantum random number generator*, SPIE Conference on Defense and Security, 2013